

A Crucial Experiment To Test The Broglie-Bohm Trajectories For Indistinguishable Particles.

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The standard quantum theory has not taken into account the size of quantum particles, the latter being implicitly treated as material points. The recent interference experiments of Zeilinger [3] with large molecules like fullerenes and the thought experiments of Bozic *et al* [7] with asymmetrical Young slits make it possible today to take into account the particle size.

We present here a complete study of this phenomenon where our simulations show differences between the particles density after the slits and the modulus square of the wave function. Then we propose a crucial experiment that allows us to reconsider the wave-particle duality and to test the existence of the Broglie-Bohm trajectories for indistinguishable particles.

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I. INTRODUCTION

The standard quantum theory of interference phenomena does not take the size of the particles into account, the latter being implicitly treated like material points. Two significant advances make this possible now thus allowing us to better apprehend the wave-particle duality.

The first advance concerns the interference experiments realized some years ago with large size mesoscopic individual quantum objects, cf. Schmiedmayer *et al* [1] and Chapman *et al* [2] with the molecules of Na_2 ($\sim 0.6nm$ size), Arndt *et al* [3] with the fullerene molecules C_{60} (~ 1 nm diameter), Nairz *et al* [4] with the molecules C_{70} and more recently Hackermüller *et al* [5] with the molecules of fluorofullerenes $C_{60}F_{48}$.

The second advance concerns the thought experiments suggested and simulated by Bozic *et al* [6, 7]: those are interference experiments with slits of various sizes, some large enough to let the molecules get through, others smaller making that impossible. It is thus theoretically possible to take into account the size of the particles [7] by studying their differences in behavior according to the respective sizes of the particles and the slits.

These thought experiments are particularly suggestive concerning the interpretation of the wave function since they correspond to cases in which *the particles density measured after the slits can be different from the calculation of the modulus square of the wave function*. It is especially the case when the particle diameter is larger than the size of all the slits and as Arndt *et al* [3] underlines it " *it would be certainly interesting to investigate the interference of objects the size of which is equal or*

even bigger than the diffraction structure". Indeed, in this case the particles density after the slits will then be null while the standard calculation of the modulus square of the wave function will not: *the postulate of the probabilistic interpretation of the modulus square of the wave function could well be questioned by this experiment and must thus be reappraised*.

The initial point of this article is to make a complete study of this phenomenon by calculating the particles density after the slits according to the various possible assumptions. The second point is to propose a crucial thought experiment to test the existence of the Broglie-Bohm trajectories for indistinguishable particles. This will be achieved by further looking into the very interesting results of Bozic *et al* [7] in two directions:

- by determining the particle "quantum trajectories" which take into account their size and the slits size. Thus one obtains the particles density after the slits which can be very different from the square of the wave function.

- by proposing some experiment making it possible to clearly highlight the difference between the density obtained by calculation of the wave function and the density obtained by the quantum trajectories.

We will make use of the experimental data of the Zeilinger team corresponding to the C_{60} molecule [3]: the molecule is spherical with a diameter of 1 nm and the slits have a width of 50 nm (it is the same size ratio as that of a soccer ball compared to the goal).

Indeed, as Bozic *et al* [7] point out, the quantum description must theoretically take into account the interaction of an extended particle with the edge of the slits. The experiment with C_{60} [3] shows that one can neglect this effect if the ratio of the sizes is rather large (~ 50 nm). This is what we will do from now on. The error could indeed come only from the particles which run up against the edges, which according to the preceding estimate would amount to an error of the order of a percent only.

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Section 2 describes a thought experiment corresponding to asymmetrical slits, then section 3 calculates the modulus square of the wave function after the slits both of this experiment and of some diffraction experiment. Then section 4 shows how the Broglie-Bohm trajectories make it possible to calculate the particles density after the slits. From that we conclude that the thought experiment suggested is a crucial experiment to test the Broglie-Bohm interpretation for indistinguishable particles.

II. THE THOUGHT EXPERIMENT

We propose a thought experiment inspired by the real experiment of Arndt *et al* [3]. One considers a molecular beam of fullerenes C_{60} , whose speed is $v_y = 200 \text{ m/s}$ along the $(0y)$ axis. Initial speeds in the other directions are considered null. The molecular beam is $7 \mu\text{m}$ wide along the $(0x)$ axis.

At $d_1 = 1$ meter of the orifice of the molecular beam a plate is placed admitting a slit A of 50 nm along the $0x$ axis and a grating B of 100 small slits of 0.5 nm of period 1 nm along the $0x$ axis. The distance between the centers of A and B is of 150 nm . The molecules of fullerenes are then observed by using a scanning laser-ionization detector placed at $d_2 = 1.25 \text{ m}$ after the slits.

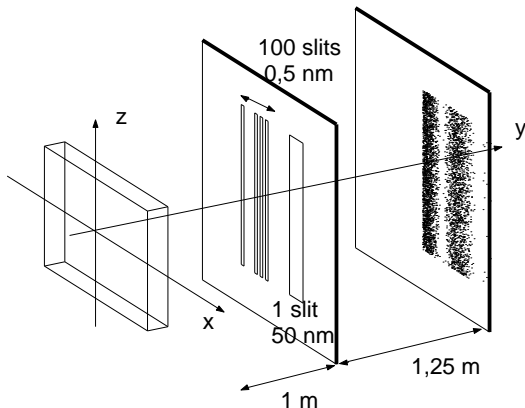


FIG. 1: Schematic diagram of the thought experiment.

Slit A corresponds to the slits of the experiment of Arndt *et al* [3]. The size of the slits of grating B was chosen to stop the molecules C_{60} . Currently, such a grating is certainly impossible to realize to day, but it will allow the thought experiment. It corresponds to the diagram of an experiment which the developments in nanotechnology should make possible. It is perhaps experimentally easier to construct a hole of 50 nm diameter in the place of slit A and a set of 10000 small holes of 0.5 nm diameter in the place of grating B; but with slits the theoretical study is clearer and the simulations are easier.

With the velocity $v_y = 200 \text{ m/s}$, the mass of the fullerene C_{60} , $m = 1.2 \times 10^{-24} \text{ kg}$ gives, a de Broglie wave-

length $\lambda_{db} = 2.8 \text{ pm}$, 350 times smaller than its diameter ($\sim 1 \text{ nm}$).

We will compare this asymmetrical slits (slit A and grating B) experiment with the diffraction experiment involving only slit A.

III. CALCULATION OF THE WAVE FUNCTION WITH FEYNMAN PATH INTEGRAL

The calculation of the wave function is obtained by a numerical calculation using the Feynman's integrals, as we did [9] for the numerical simulation of the experiment of the slits of Shimizu *et al* with cold atoms.[10]

For the numerical simulation, we make the following assumptions. The slits being very long along the $0z$ axis, there is no diffraction according to this axis, but the particles are subjected to gravity along $(0z)$. Consequently, the variable z can be treated classically as the variable y , satisfying the relations $y = v_y t$ and $z = -\frac{1}{2}gt^2$. We thus consider only the wave function in x , $\psi(x, t)$; we take as initial wave function $\psi_0(x) = (2\pi\sigma_0^2)^{-\frac{1}{4}} \exp^{-\frac{x^2}{4\sigma_0^2}}$ with $\sigma_0 = 2 \mu\text{m}$.

The wave function before the slits is then equal to

$$\psi(x, t) = (2\pi s(t)^2)^{-\frac{1}{4}} \exp^{-\frac{x^2}{4\sigma_0 s(t)}} \quad (1)$$

with $s(t) = \sigma_0(1 + \frac{i\hbar t}{2m\sigma_0^2})$. After the slits, at time $t \geq t_1 = \frac{d_1}{v_y} = 5 \text{ ms}$, we use the Feynman path integral method to calculate the time-dependent wave function [16] :

$$\psi(x, t) = \int_F K(x, t; x_f, t_1) \psi(x_f, t_1) dx_f \quad (2)$$

where

$$K(x, t; x_f, t_1) = \left(\frac{m}{2i\pi\hbar(t-t_1)}\right)^{\frac{1}{2}} \exp \frac{im}{\hbar} \frac{(x-x_f)^2}{2(t-t_1)} \quad (3)$$

and where integration in (2) is carried out on set F of the area of the various slits and where $\psi(x_f, t_1)$ is given by (1).

On figure 2 is represented the modulus square of the wave function on the detection screen of (a) the diffraction experiment with slit A only, and (b) the interference experiment with asymmetrical slits (slit A and grating B). Figure 2c is just a zoom on central part of figure 2b. We note that 20 percent of the total density is not represented on figure 2c, as one part moves laterally towards the right of the screen and another part leftwards. For the asymmetrical slits experiment, the modulus square of the wave function is asymmetrical in the first centimeters after the slits, then becomes fairly symmetrical when it comes to the detection screen placed at 1.25 meter (figures 2b and 2c).

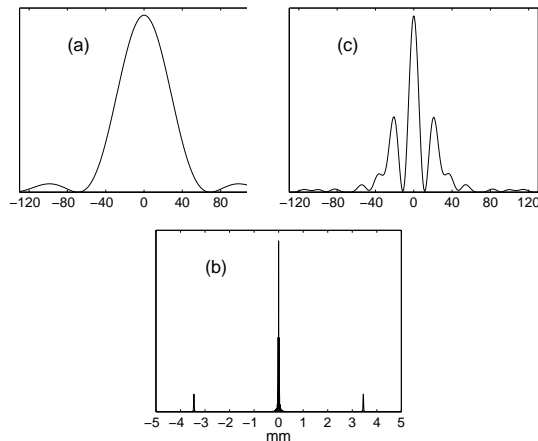


FIG. 2: Modulus square of the wave function on the detection screen respectively: (a) diffraction (slit A), (b) interference with asymmetrical slits (slit A and grating B), (c) zoom of the asymmetric slits central part.

The standard interpretation of quantum mechanics postulates that the density of the particles must be equal to the modulus square of the wave function. The particles density on the detection screen of the asymmetrical slits experiment must thus be given by figures 2b and 2c (Standard Assumption 1): we obtain three peaks on figure 2b (a central peak and two little peaks at -3,4 mm and 3,3 mm) and also three peaks on figure 2c (a central peak and two smaller peaks at -20 μm and +20 μm). If it is supposed that the molecules cannot pass through grating B, the density shown on figures 2b and 2c can then be in contradiction with the experimental results. However within the framework of standard interpretation, another solution is possible: one can make the assumption that if the particles do not pass through grating B, then the wave function does not pass through there either (Standard Assumption 2). The experimental result must then be given by figure 2a of the diffraction with the single slit A which corresponds now to one unique peak.

IV. CALCULATION OF THE PARTICLES DENSITY WITH BROGLIE-BOHM TRAJECTORIES

In the Young slits experiments the interference fringes, and thus the wave function, are never observed directly. The only direct measurements are the individual impacts of the particles on the detection screen. In the Broglie-Bohm interpretation, [11, 12, 13] the particle is represented not only by its wave function, but also by the position of its center of mass. The center of mass of the particle follows a trajectory, which is piloted by the wave function ψ with a speed \mathbf{v} given by

$$\mathbf{v} = \frac{\hbar}{2m\rho} \text{Im}(\psi^\dagger \nabla \psi). \quad (4)$$

This interpretation statistically gives, in all the examples available in the literature, the same experimental results as the Copenhagen interpretation. Moreover, the Broglie-Bohm interpretation naturally explains the individual impacts. These impacts correspond, as in classic mechanics, with the position of the particles at the time of their arrival on the detection screen.

The experiment with asymmetrical slits corresponds to a case where the Copenhagen and Broglie-Bohm interpretations give different results. It is then possible to test the Broglie-Bohm Assumption in a very simple manner. One simply has to simulate the Broglie-Bohm trajectories and to compare these simulation results with the experiment.

In the standard interpretation, a molecule C_{60} is represented at the initial time only by its wave function $\psi_0(x) = (2\pi\sigma_0^2)^{-\frac{1}{4}} \exp\left(-\frac{x^2}{4\sigma_0^2}\right)$. It thus has an uncertain initial position since the wave function only gives the

probability density $\rho_0(x) = (2\pi\sigma_0^2)^{-\frac{1}{2}} \exp\left(-\frac{x^2}{2\sigma_0^2}\right)$. Since the particle is indistinguishable, at time t one can only know its probability density. Such indistinguishable particles can also be found in classic mechanics, when one only knows speed and initial probability density: to describe the evolution of such classic particles and to make them distinguishable, it is necessary to know their initial positions. In quantum mechanics, de Broglie and Bohm make the same assumption for indistinguishable particles.

As we did for cold atoms[9], we set up a Monte Carlo simulation of the experiment by randomly drawing the initial positions for molecules C_{60} in the initial wave function.

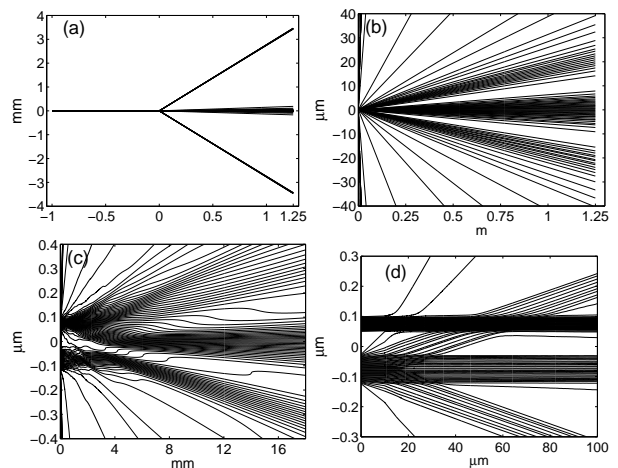


FIG. 3: 100 Broglie-Bohm trajectories with randomly drawn initial positions: (a) global view, (b) central trajectories, (c) zoom on the first millimeters after the slits, (d) zoom on the hundred first micrometers after the slits.

In the first place, one does not take into account the C_{60} size. Figure 3 represents the quantum trajectories of 100 molecules C_{60} which cross either slit A or grating B (one did not represent the molecules stopped by the

plate). The density of these molecules on the detection screen corresponds to the modulus square of the wave function represented on figures 2b and 2c. On figure 3d we retrieve the loss of 20 percent of the density in the trajectories, which accounts for particles which move laterally. At a second stage one takes into account the fact that all molecules C_{60} are stopped by grating B. Figure 4 shows the quantum trajectories of molecules C_{60} which pass slit A only.

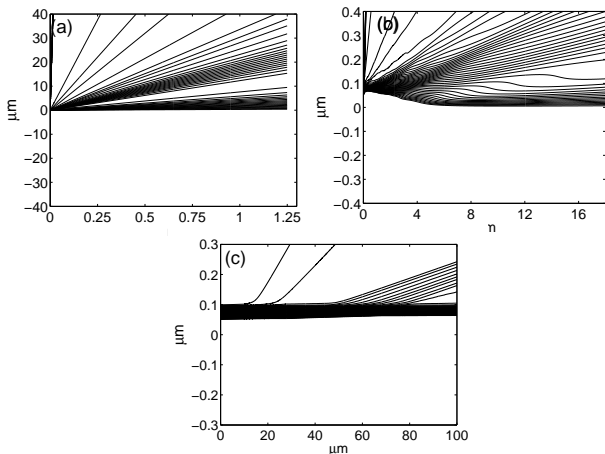


FIG. 4: Broglie-Bohm trajectories through slit A only: (a) global view of trajectories, (b) zoom on the first millimeters, (c) zoom on the hundred first micrometers.

Figure 5 shows the density on the detection screen of molecules C_{60} which pass slit A only. The density is to be found experimentally if the Broglie-Bohm assumption is valid.

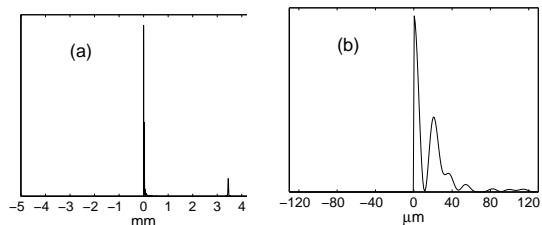


FIG. 5: Density of molecules C_{60} having crossed slit A: (a) global density, (b) central density.

The clear difference between the density of Standard Assumption 1 (figures 2b and 2c with three peaks), Standard Assumption 2 (figure 2a with one peak) and the Broglie-Bohm Assumption (asymmetrical figures 5a and 5b with two peaks only) shows that it is possible to define a robust test in spite of certain experimental difficulties:

- in preceding calculations we supposed that initial speeds were well-known. It is not the case in the experiments of Zeilinger where for example speed v_y is not well-known. We have shown [9] how to take these uncertainties into account. They will smooth the densities, but will preserve the number of peaks.

- to ensure the validity of the calculation of wave function (2), it is necessary to prevent molecules C_{60} from being blocked in the slits of grating B and stopped there. For that purpose, one can slightly incline the plate so that the particles rebound while falling downwards. One can also send them one by one in order to leave to those which are stopped enough time to fall before the arrival of the following particles. One can also make slits even narrower.

V. CONCLUSION

Taking into account the size of the particles in the interference phenomena, makes it possible to renew the study of the wave-particle duality and to propose experiments to test the Broglie-Bohm trajectories for indistinguishable particles. Should the test be positive, the wave function for indistinguishable particles would have to be considered as a field. For the distinguishable particles, this interpretation does not apply [14].

While waiting for the results of this test which raises experimental difficulties, one can already realize a much simpler experiment which makes it possible to clarify the standard postulate of the probabilistic interpretation of the modulus square of the wave function: it is the test of Standard Assumption 2 against Standard Assumption 1. This can be achieved experimentally by simply reducing the size of slits and holes to less than the diameter of molecule C_{60} . That should suffice to observe that the density on the detection screen is null.

Professor Bozic has suggested that our thought experiment might be carried out with Rydberg atoms instead of C_{60} molecules. Indeed Fabre et al [15] have shown that slits of $1 \mu m$ are enough to stop Rydberg atoms as soon as $n = 50$. The experiment would then be performable right now.

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- Verlag, Berlin, 1993), p.21.
- [2] M.S. Chapman, C.R. Ekstrom, T. D. Hammond, R.A. Rubenstein, J. Schmiedmayer, S. Wehinger, and D. E. Pritchard, Phys. Rev. Lett. **74**, 4783 (1995).
 - [3] M. Arndt, O. Nairz, J. Voss-Andreae, C. Keller, G. van des Zouw, and A. Zeilinger, "Wave-particle duality of C60 molecules", Nature (London) **401**, 680–682 (1999).
 - [4] O. Nairz, M. Arndt, and A. Zeilinger, "Experimental challenges in fullerene interferometry", J. Mod. Opt. **47**, 2811–2821 (2000).
 - [5] L. Hackermüller, S. Uttenthaler, K. Hornberger, E. Reiger, B. Brezger, A. Zeilinger, and M. Arndt, "The wave nature of biomolecules and fluorofullerenes", Appl. Phys. B Lasers Opt. **77**, 781 (2003).
 - [6] M. Bozic, D. Arsemoric, and L. Vuskovic, Bull. Am. Phys. Soc. **47**, 74 (2002).
 - [7] M. Bozic, D. Arsemoric, and L. Vuskovic, "Interference of small and of large quantum particles behind an asymmetric grating", Phys. Rev. A **69**, 053618 (2004).
 - [8] H. Rauch and A. Werner, *Neutron Interferometry : Lessons in Experimental Quantum Mechanics* (Oxford Univ. Press, London, 2000).
 - [9] M. Gondran, and A. Gondran, Am. J. Phys. **73**, 507-515 (2005).
 - [10] F. Shimizu, K. Shimizu, and H. Takuma, Phys. Rev. A **46**, R17-R20 (1992).
 - [11] L. de Broglie, "La mécanique ondulatoire et la structure atomique de la matière et du rayonnement," J. de Phys. **8**, 225–241 (1927); L. de Broglie, *Une Tentative d'Interprétation Causale et Non Linéaire de la Mécanique Ondulatoire* (Gauthier-Villars, Paris, 1951).
 - [12] D. Bohm, "A suggested interpretation of the quantum theory in terms of 'hidden' variables", Phys. Rev. **85**, 166–193 (1952).
 - [13] D. Bohm and B. J. Hiley, *The Undivided Universe* (Routledge, London and New York, 1993); P. R. Holland, *The Quantum Theory of Motion* (Cambridge University Press, 1993). Presentation of the de Broglie-Bohm interpretation of quantum mechanics equations.
 - [14] M. Gondran, and A. Gondran, "Mécanique quantique : particules indiscernables et interprétation statistique", REE **2**, feb. 2006.
 - [15] C. Fabre, M. Gross, J.M. Raimond, and S. Haroche, J. Phys. B **16**, L671 (1983).
 - [16] R. Feynman and Hibbs, *Quantum Mechanics and Integrals* (McGraw-Hill, Inc., 1965) p.41-64.